END
DATE
FILMED
1 83
DTIC

MICROCOPY RESOLUTION TEST CHART

NATIONAL BUREAU OF STANDARDS-1963-A

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LINCOLN LABORATORY

CONSUMPTION OF DEGREES OF FREEDOM
IN ADAPTIVE NULLING ARRAY ANTENNAS

A.J. FENN

Group 61

TECHNICAL REPORT 609

12 OCTOBER 1982

LEXINGTON   MASSACHUSETTS

Abstract

A gradient search technique is used to maximize consumption of the degrees of freedom for N-channel adaptive nulling phased array antennas. The technique is based on a figure of merit which seeks to maximize the sum of the square roots of the interference covariance matrix eigenvalues. Equivalently, the worst case interference source configuration attempts to completely consume N degrees of freedom of the adaptive nulling antenna. Results are given for several basic regularly spaced arrays. The technique can be applied to a phased array with arbitrary array element positions.

CONTENTS

CONTENTS (cont'd)

ILLUSTRATIONS

TABLES

I. INTRODUCTION

The main goal of an adaptive nulling antennna used in any communications
system is to reduce the received power from undesired interference sources
while maintaining sufficient pattern gain (or signal-to-noise ratio) to
desired sources in the antenna field of view. Generally, if a single
interference signal is sensed by an adaptive system, the amplitude and phase
responses of the antenna elements are adjusted so that a pattern null is
formed in the interference direction. Assuming that the desired sources are
located in directions sufficiently far away from the null directions,
communications will not be disrupted.

A typical adaptive nulling antenna system block diagram is shown in
Fig. 1.1. In general, an N-element array or N-beam multiple beam antenna
(MBA) has N-degrees of freedom. That is, there are N adjustable weights,
$w_1, w_2, \ldots, w_N$, which can be set to synthesize a desired radiation pattern at K
far-field points. This means that, in general, N-1 pattern minima (or nulls)
can be synthesized while simultaneously maintaining some antenna gain in the
direction of a desired source. Clearly, if there are N-1 interference sources
distributed in the antenna field of view, then N-1 distinct nulls can be used
to suppress them.

A situation where the interference potentially can be more severe is the
case of N interference sources as shown in Fig. 1.2. Depending on the antenna
configuration, the above discussion suggests that N interference sources
possibly could consume all N-degrees of freedom. Since only N-1 nulls are
available, the Nth source may tend not to be nulled. In fact, if N-sources

1
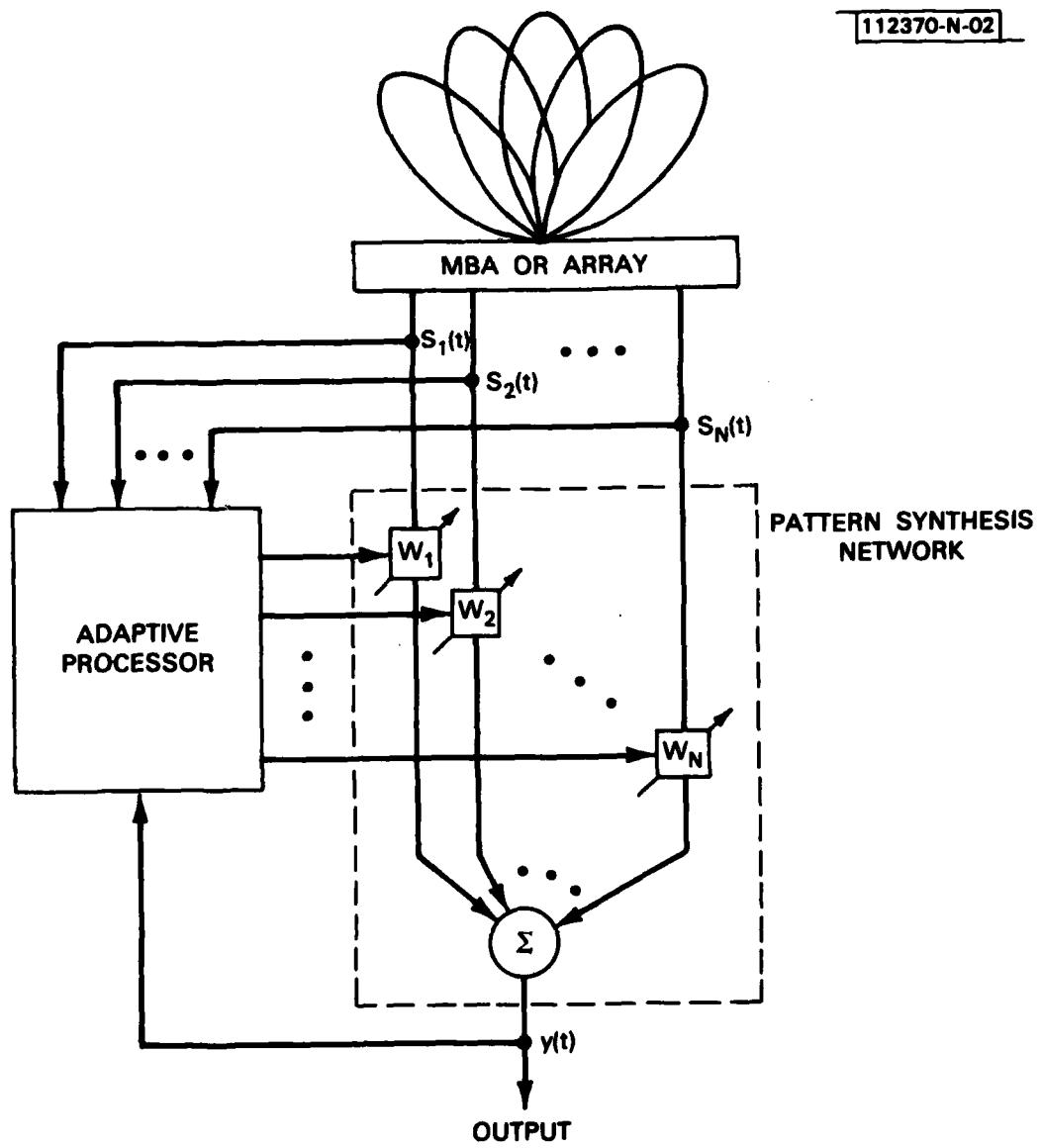
Figure 1.1 Block diagram for an N-channel adaptive nulling system.

2
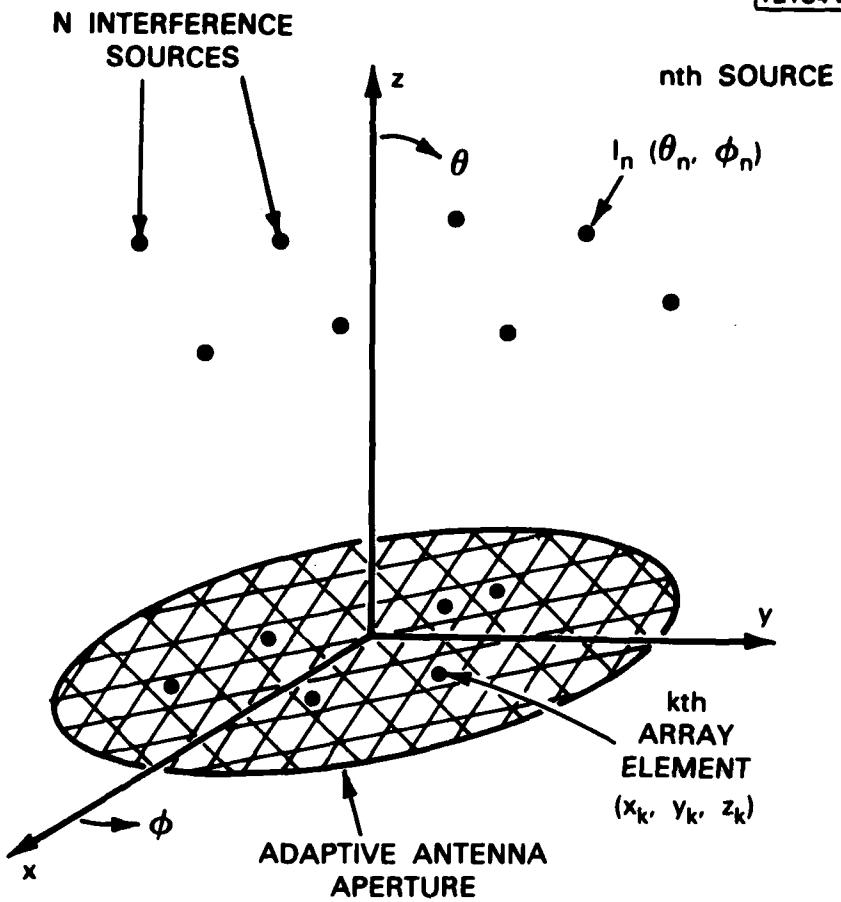
Figure 1.2 Distribution of N inteference sources in the field of view of an arbitrary adaptive nulling system.

completely consume N-degrees of freedom of an N-channel antenna, no useful adaptive nulling is possible.

An important question is: does a geometric configuration of N sources exist that completely consumes N-degrees of freedom? The case of two and three sources in the field of view of a phased array and a multiple beam antenna has been studied and the results described in a previous report[1]. The consumption of degrees of freedom was found to be dependent on both source spacing and source configurations. The spacing should be approximately one half-power beamwidth in order that each source consumes one degree of freedom. However, the worst case interference configuration depends on the nulling antenna geometry.

To this author's knowledge, a general method for the analytical determination of the worst case N-interference source geometrical configuration has not previously been developed. It is the intention of this report to do so.

In order to fully understand the concept of adaptive antenna degrees of freedom, it is helpful to analyze the covariance matrix formed by the cross-correlation of interference signals received at each pair of antenna elements. In Section II, this interference covariance matrix is expressed in terms of its eigenvalues and eigenvectors. The conditions necessary for complete consumption of the antenna degrees of freedom are determined.

In Section III, a derivation of the interference signal matrix which consumes N-degrees of freedom of an arbitrary adaptive nulling array antenna is given. The interference signal matrix is shown to be unitary; that is,

4

the incident signal vectors are orthonormal. This leads to the concept of orthogonal interference sources which is discussed in Section IV. An equation is derived which gives the N-interference source configuration which completely consumes N-degrees of freedom of an N-element equally-spaced linear array. This is demonstrated for an 8-element array with eight interference sources.

One of the interesting results of this study is that for some N-element array antenna configurations, N-degrees of freedom cannot be completely consumed by N sources. Section V discusses this case, and a source figure of merit is derived which can be used to determine the worst case N-source configuration. The figure of merit is equal to the sum of the square roots of the eigenvalues computed from the interference covariance matrix. The most effective N-source configuration occurs when the figure of merit is a maximum. That is, consumption of antenna degrees of freedom is maximized.

In Section VI, a numerical gradient search technique (based on the above figure of merit) is developed which finds the worst-case geometrical configuration of N interference sources. This numerical technique is useful for finding the worst-case interference source locations for arrays in which an analytic solution is difficult or impossible. Examples are given where the worst-case source configuration of various arrays is found by using the gradient search.

II. CONSUMPTION OF ADAPTIVE ANTENNA DEGREES OF FREEDOM

In this section, conditions are given for N-interference sources to completely consume N-degrees of freedom of an N-channel adaptive nulling antenna. For discussion purpose, the adaptive nulling algorithm is assumed to be the Applebaum-Howells analog servo-control-loop processor[2,3,4]. However, the results are expected to apply equally well to any adaptive nulling algorithm.

A. Applebaum-Howells Analog Servo-Control-Loop Processor

For this algorithm the steady-state adapted antenna weight column vector is given by

$$\underline{w} = [\underline{\underline{I}} + \mu \underline{\underline{R}}]^{-1} \underline{w}_0 \qquad (2.1)$$

where $\underline{\underline{I}}$ is the identity matrix

$\underline{\underline{R}}$ is the channel covariance matrix

μ is the effective loop gain which provides the threshold for sensing signals

$\underline{w}_0$ is a weight vector which gives a desired quiescent radiation pattern in the absence of interference sources.

Note: The double underbar ($\underline{\underline{\ }}$) refers to square matrix and the single underbar ($\underline{\ }$) refers to a column matrix.

For an N-channel adaptive nulling processor $[\underline{\underline{I}} + \mu \underline{\underline{R}}]$ is an N x N matrix. The covariance matrix elements are defined by

6

$$R_{pq} = \frac{1}{\omega_2 - \omega_1} \int_{\omega_1}^{\omega_2} S_p(\omega) \, S_q^*(\omega) \, d\omega \qquad \begin{array}{l} p = 1,2,\ldots,N \\ q = 1,2,\ldots,N \end{array} \qquad (2.2)$$

where $\omega_2 - \omega_1$ is the nulling bandwidth

$S_p(\omega)$, $S_q(\omega)$ are the received voltages in the pth and q^{th} channels, respectively

*   denotes complex conjugate.

Equation (2.1) is all that is needed to predict the steady-state adaptive nulling performance of the system. However, to understand how N-interference sources consume N-degrees of freedom, it is necessary to express the adapted weight vector in terms of the eigenvalues and eigenvectors of the interference covariance matrix.

B. Eigenvalues and Eigenvectors of the Interference Covariance Matrix

The covariance matrix defined by Eq. (2.2) is Hermitian (that is, $\underline{R} = \underline{R}^{\dagger}$ where $\dagger$ means complex-conjugate-transposed) which by the spectral theorem can be decomposed in eigenspace as[5]

$$\underline{R} = \sum_{k=1}^{N} \lambda_k \, \underline{e}_k \, \underline{e}_k^{\dagger} \qquad (2.3)$$

where λ_k, $k=1,2,\ldots,N$ are the eigenvalues of $\underline{R}$

$\underline{e}_k$, $k=1,2,\ldots,N$ are the associated eigenvectors of $\underline{R}$.

The matrix product $\underline{e}_k \, \underline{e}_k^{\dagger}$ is an NxN matrix which represents the projection onto eigenspace for λ_k. Comparing Eq. (2.2) and Eq. (2.3), it is observed

7

that the eigenvalues have units of voltage squared, that is, the eigenvalues are proportional to power.

Substituting Eq. (2.3) into Eq. (2.1) and using the orthogonality property of the eigenvectors leads to the following expression for the adapted antenna weight vector[1].

$$\underline{w} = \underline{w}_o - \sum_{k=1}^{N} \frac{\mu\lambda_k}{1 + \mu\lambda_k} \langle \underline{e}_k\dagger, \underline{w}_o \rangle \underline{e}_k \qquad (2.4)$$

where $\langle \underline{e}_k\dagger, \underline{w}_o \rangle = \underline{e}_k\dagger \cdot \underline{w}_o$ is a complex scalar.

Each of the vectors in Eq. (2.4) are weights which can be applied t the adaptive antenna. From Eq. (2.4), and using the principle of superposition, the adapted far-field pattern can be written as

$$P(\theta,\phi;\underline{w}) = P_o(\theta,\phi;\underline{w}_o) - \sum_{k=1}^{N} \frac{\mu\lambda_k}{1 + \mu\lambda_k} \langle \underline{e}_k\dagger, \underline{w}_o \rangle P_k(\theta,\phi;\underline{e}_k) \qquad (2.5)$$

where $P(\theta, \phi; \underline{w})$ is the adapted radiation pattern

$P_o(\theta, \phi; \underline{w}_o)$ is the quiescent radiation pattern

$P_k(\theta, \phi; \underline{e}_k)$ is the kth eigenvector radiation pattern.

Equation (2.5) shows how the quiescent radiation pattern is modified in the presence of interference sources. The scalar $\langle \underline{e}_k\dagger, \underline{w}_o \rangle$ is the projection of the kth eigenvector onto the quiescent antenna weight vector. If, for example, a single interference source (which gives rise to a single eigenvalue, λ_1, and eigenvector, $\underline{e}_1$) lies on a null of the quiescent pattern, then $\langle \underline{e}_1\dagger, \underline{w}_o \rangle = 0$. No adaption is necessary which means that $\underline{w}=\underline{w}_o$. However, if

8

a source lies on a sidelobe of the quiescent pattern, the product of $\underline{e}_1$† with $\underline{w}_o$ will be non-zero. This projection is weighted by the quantity $\mu\lambda_1/(1 + \mu\lambda_1)$ and subtracted from $\underline{w}_o$. For a large value of λ_1 corresponding to a strong interference source, the product $\mu\lambda_1$ is much greater than unity. This implies that $\mu\lambda_1/(1 + \mu\lambda_1)\cong 1$. (Similarly, a weak interference source which has $\mu\lambda_1$ much less than unity results in $\mu\lambda_1/(1 + \mu\lambda_1)\mu \cong 0$.) If another source sufficiently separated (by approximately one half-power beamwidth) from the first is added, a second large eigenvalue, λ_2, will occur[1]. Two terms ($k=1,2$) would then be significant in Eq. (2.5).

From the above examples, it is clear that strong sources cause a larger change in the quiescent weight vector than do weak sources. However, this is only true when the number of degrees of freedom is sufficient to null the interference. In the next section, it is shown that if N-degrees of freedom are completely consumed, the quiescent weight vector does not change after adaption. In this case, interference signals will not be adaptively nulled.

C. <u>Definition of Complete Consumption of N-Degrees of Freedom</u>

N-degrees of freedom of an N-channel adaptive nulling antenna are consumed when the following conditions are met:

1. There are N uncorrelated equal-power interference sources in the antenna field of view.

2. The interference power is large enough to be sensed by the nulling system.

3. The antenna fails to form any adaptive nulls in the interference source directions.

To explain this set of conditions, we first note that it may be inferred from a previous study that to consume N-degrees of freedom, the interference covariance matrix must have N large eigenvalues[1]. This will tend to occur when the sources are spaced on the order of the antenna half-power beamwidth.

Given N equal-power uncorrelated sources arranged such that the interference covariance matrix has N large equal eigenvalues, then in Eq. (2.4)

$$\frac{\mu\lambda_k}{1 + \mu\lambda_k} = C = \text{Constant} \quad . \tag{2.6}$$

Thus, Eq. (2.4) can be written as

$$\underline{w} = \left[\underline{\underline{I}} - C \sum_{k=1}^{N} \underline{e}_k \underline{e}_k^\dagger\right]\underline{w}_o \quad . \tag{2.7}$$

It can be shown that[11]

$$\sum_{k=1}^{N} \underline{e}_k \underline{e}_k^\dagger = \underline{\underline{I}} \tag{2.8}$$

so

$$\underline{w} = (1 - C)\underline{w}_o \quad . \tag{2.9}$$

That is, when N sources are arranged so as to create an interference covariance matrix with N large, equal eigenvalues, then N-degrees of freedom

10

are completely consumed, and the adapted weight vector is equal to a constant times the quiescent weight vector. This means that the antenna radiation pattern (as given by Eq. (2.5)) cannot change from its quiescent shape. In other words, interference sources on the antenna pattern sidelobes cannot be nulled.

D. Discussion

In section (II.C), the basic conditions necessary for complete consumption of antenna degrees of freedom were given. A fundamental mathematical condition for this occurrence requires that the interference covariance matrix possess N large (compared to quiescent receiver noise) equal eigenvalues.

It is shown in the next section that when the covariance matrix has N large equal eigenvalues, it is a diagonal matrix. Furthermore, the interference signal matrix is shown to be unitary. For convenience, the derivation is given for an adaptive phased array antenna. However, the same results should apply to a multiple-beam antenna.

III. DERIVATION OF INTERFERENCE SIGNAL MATRIX TO CONSUME N DEGREES OF FREEDOM OF AN N-CHANNEL ADAPTIVE NULLING ARRAY ANTENNA

A. Introduction

In this section, a mathematical proof is given which shows that the optimum configuration of N equal power uncorrelated interference sources which consumes the N degrees of freedom of an N-channel adaptive nulling array antenna, requires a unitary signal matrix. The NxN covariance matrix which is formed from the received interference signals in this case is a diagonal matrix.

The concept of "orthogonal sources" has been discussed by Mayhan, et. al[4]. By definition, each of these sources consume a complete degree of freedom. A necessary condition is that the sources be separated angularly by approximately one half-power beamwidth[1]. The received signal matrix for orthogonal sources has orthogonal columns. This leads to covariance matrix eigenvalues which are proportional to the incident interference power. This will be shown in the following paragraphs.

B. Derivation

With little loss of generality, it can be assumed that the nulling antenna is an array of isotropic point sources. The elements are assumed to be located in the xy plane of the rectangular coordinate system such that the kth element has arbitrary coordinates (x_k, y_k). Further, it is assumed that the ith interference source with incident power P_i is located at a large distance from the array, at angles (θ_i, ϕ_i) where θ and ϕ are standard spherical coordinates. Then the $ki\underline{th}$ element of the received signal matrix (denoted by $\underline{S}$) is given by

$$S_{ki} = \sqrt{P_i} \ e^{j\pi D/\lambda \ \sin\theta_i (x_k \cos\phi_i + y_k \sin\phi_i)} \qquad \begin{array}{l} k=1,2,\ldots,N \\ i=1,2,\ldots,N \end{array} \qquad (3.1)$$

where D is the array diameter and $x_k^2 + y_k^2 \leq 1$ are the normalized element positions (relative to $D/2$).

The covariance matrix for narrowband interference is

$$\underline{R} = \underline{S} \ \underline{S}^\dagger + \underline{I} \qquad (3.2)$$

where $\dagger$ means complex-conjugate transposed, and

$\underline{I}$ is the identity matrix which is used here to effectively normalize the covariance matrix to quiescent receiver noise, that is, P_i is measured relative to receiver noise.

The above covariance matrix is Hermitian (that is, $\underline{R} = \underline{R}^\dagger$), so it can be decomposed in eigenspace as (see Eq. 2.3)

$$\underline{R} = \sum_{n=1}^{N} \lambda_n \ \underline{e}_n \ \underline{e}_n^\dagger \qquad (3.3)$$

where λ_n is the nth eigenvalue of $\underline{R}$

$\underline{e}_n$ is the nth eigenvector of $\underline{R}$.

To better understand Eq. (3.3), consider the following cases:

Assume first that interference sources are not present, that is, $P_i = 0$, ($i = 1, 2, \ldots, N$) so that $\underline{S} = [0]$ is the null matrix and $\underline{R} = \underline{I}$. For this case

$$\lambda_1 = \lambda_2 = \ldots = \lambda_N = \lambda_q = 1$$

where λ_q is the eigenvalue due to receiver noise only.

This is true because the sum of the diagonal elements of $\underline{R}$ equals the sum of the eigenvalues of $\underline{R}$, that is[12]

$$\sum_{i=1}^{N} R_{11} = \sum_{n=1}^{N} \lambda_n = \sum_{n=1}^{N} 1 = N \quad . \tag{3.4}$$

Assume now that N large equal-power interference sources are present, thus

$$P_1 = P_2 = \ldots = P_N = P \quad ,$$

where P is the power from each interference source received at each array element. These sources produce eigenvalues which are large compared to quiescent receiver noise, that is, $\lambda_m \gg \lambda_q$. To completely consume N degrees of freedom requires the covariance matrix to have N identical eigenvalues*, such that for an array

$$\lambda_1 = \lambda_2 = \ldots = \lambda_N = \lambda = (PN + 1) \tag{3.5}$$

That is, the eigenvalues are proportional to the incident interference power. A proof of Eq. (3.5) is given in Appendix A. Substituting this result in Eq. (3.3)[†] yields

*Examples will be given later that demonstrate, for N equal eigenvalues, the antenna cannot provide any adaptive pattern nulling.

[†]We assume for the moment that we can find an "optimum" source distribution that will result in Eq. 3.5 being satisfied. Later we show that this ideal distribution can in general only be approximated.

14

$$\underset{\Xi}{R} = \lambda \sum_{n=1}^{N} \underline{e}_n \underline{e}_n^\dagger \quad . \qquad (3.6)$$

From Eq. (2.8)

$$\sum_{n=1}^{N} \underline{e}_n \underline{e}_n^\dagger = I \qquad (3.7)$$

Thus,

$$\underset{\Xi}{R} = \lambda \underset{\Xi}{I} = (PN + 1)\underset{\Xi}{I} \qquad (3.8)$$

which says that when N degrees of freedom are consumed, the covariance matrix is diagonalized with N equal eigenvalues on the diagonal.

Substituting Eq. (3.8) in Eq. (3.2) yields, for the assumed optimum source distribution

$$\underset{\Xi}{S} \underset{\Xi}{S}^\dagger = (\lambda-1)\underset{\Xi}{I} \quad . \qquad (3.9)$$

Let

$$\underset{\Xi}{S}_n = \frac{\underset{\Xi}{S}}{\sqrt{\lambda-1}} = \frac{\underset{\Xi}{S}}{\sqrt{PN}} \qquad (3.10)$$

denote the normalized optimum signal matrix. Substituting this in Eq. (3.9) yields

$$\underset{=n}{S} \underset{=n}{S}^\dagger = \underset{=}{I} \qquad . \qquad\qquad\qquad (3.11)$$

By definition, a matrix U is unitary if

$$\underset{=}{U} \underset{=}{U}^\dagger = \underset{=}{I} \qquad\qquad\qquad\qquad (3.12)$$

Equation (3.11) satisfies Eq. (3.12) and thus $\underset{=n}{S}$ is unitary. This result shows that the covariance matrix formed from a unitary signal matrix is equal to the identity matrix.

C. Discussion

A mathematical derivation is given in this section which, as a result of Eq. (3.11) proves that when the N degrees of freedom of an N channel adaptive nulling array antenna are completely consumed by N equal-power interference sources, a unitary signal matrix is produced. (Although it is not shown, it is expected that the same result should be valid for a multiple beam antenna.) The optimum signal matrix $\underset{=}{S}$ (which is equal to the unitary signal matrix $\underset{=n}{S}$ multiplied by the square root of the product of the incident interference power and the number of elements (Eq. (3.10)) was shown to produce a diagonalized covariance matrix with N equal eigenvalues on the diagonal (Eq. 3.8). The eigenvalues were shown in Appendix A to be proportional to the incident interference power.

While the above result shows that a unitary signal matrix is optimum (to consume degrees of freedom), it does not (in general) provide the interference

16

source locations directly, and such a configuration may not exist for an arbitrary antenna layout.   Sections V and VI discuss an iterative best approximation to a unitary signal matrix, which determines N interference source locations.   For certain special cases, the optimum signal matrix can be achieved, however.   In the next section, an exact solution for the optimum location of N interference sources in the field of view of an equally spaced N element linear array is obtained.

IV. CONFIGURATION OF N INTERFERENCE SOURCES TO CONSUME N DEGREES-OF-FREEDOM OF AN N-ELEMENT ADAPTIVE NULLING LINEAR ARRAY ANTENNA

A. Introduction

In Section III it was shown that the optimum configuration of N interference sources, to consume N degrees-of-freedom of an N-channel adaptive nulling array antenna, produces a unitary signal matrix. For arrays with elements located on a periodic lattice, the worst-case interference source configuration that consumes N degrees-of-freedom can sometimes be computed analytically. (In general, this is not possible for random, thinned, or irregular arrays.) In this section, the optimum interference source coordinates θ_i, $i=1,2,\ldots,N$, for a linear array, are derived by enforcing the orthogonality property of the optimum covariance matrix (Eq. 3.8). Following the derivation, an example, consisting of a configuration of eight interference sources, to consume eight degrees-of-freedom of an eight-element linear array, is given.

B. Derivation of Orthogonal Interference Source Positions

Consider a linear array of N equally-spaced isotropic elements as shown in Figure 4.1. It is desired to compute the angular positions θ_i, $i=1,2,\ldots,N$ of N interference sources such that N degrees-of-freedom are completely consumed. From Section III, this occurs when the interference signal matrix is unitary and, equivalently, when the normalized covariance matrix is equal to the identity matrix.

The received signal at array element k, due to the ith interference source is given as
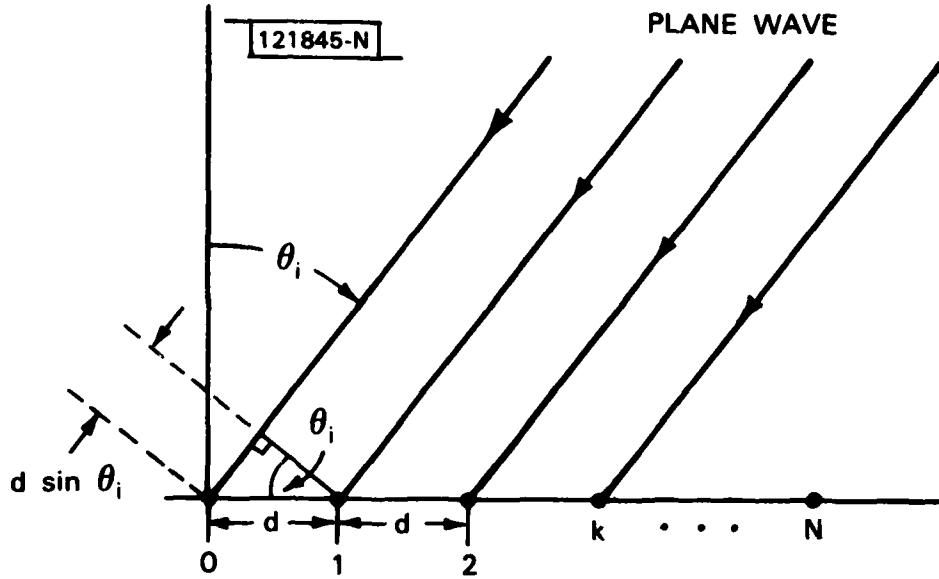
$$S_{ki} = e^{j2\pi kd/\lambda \; \sin \theta_i}$$

$$k = 0,\; 1,\; 2,...,\; N-1$$

Figure 4.1 Received signal at the kth element of an N-element linear array due to the ith interference source.

$$S_{ki} = e^{j2\pi kd/\lambda \sin \theta_i} \qquad \begin{array}{l} k = 0,1,2,\ldots,N-1 \\ i = 1,2,\ldots,N \end{array} \qquad (4.1)$$

where λ is the wavelength (which should not be confused with the eigenvalue λ).

The normalized covariance matrix for narrowband interference is defined here to be

$$\underline{\underline{K}}' = \underline{\underline{S}} \; \underline{\underline{S}}^{\dagger} \qquad (4.2)$$

where $\underline{\underline{S}}$ is the signal matrix, $\dagger$ means complex-conjugate transposed. (Note: to simplify the derivation, the identity matrix is dropped (see Eq. 3.2)).

Let $\psi_i = 2\pi d/\lambda \sin \theta_i$; then from Eq. (4.1), the interference signal matrix is

$$\underline{\underline{S}} = \begin{bmatrix} 1 & 1 & & \cdot & \cdot & \cdot & 1 \\ e^{j\psi_1} & e^{j\psi_2} & & & & & e^{j\psi_N} \\ e^{j2\psi_1} & e^{j2\psi_2} & & & \cdot & & e^{j2\psi_N} \\ \vdots & \vdots & & & & \cdot & \vdots \\ e^{j(N-1)\psi_1} & e^{j(N-1)\psi_2} & & & & & e^{j(N-1)\psi_N} \end{bmatrix}$$

$$(4.3)$$

and

$$S^{\dagger} = \begin{bmatrix} 1 & e^{-j\psi_1} & e^{-j2\psi_1} & \cdot & \cdot & \cdot & e^{-j(N-1)\psi_1} \\ 1 & e^{-j\psi_2} & e^{-j2\psi_2} & \cdot & \cdot & \cdot & e^{-j(N-1)\psi_2} \\ \vdots & & & \cdot & \cdot & \cdot & \\ \vdots & & & & \cdot & & \vdots \\ 1 & e^{-j\psi_N} & e^{-j2\psi_N} & \cdot & \cdot & \cdot & \cdot & e^{-j(N-1)\psi_N} \end{bmatrix}$$

$$(4.4)$$

20

Using Eqs. (4.2), (4.3), and (4.4), the mn^{th} term of the covariance matrix is expressed as

$$K'_{mn} = \sum_{i=1}^{N} e^{j(m-n)\psi_i} \qquad \begin{array}{l} m = 0,1,2,\ldots,N-1 \\ n = 0,1,2,\ldots,N-1 \end{array} \qquad (4.5)$$

(Note: $\underline{K}$ is a Hermitian matrix since $K_{mn} = K_{nm}^*$ (* denotes complex conjugate). Enforcing orthogonality of the covariance matrix (to consume N degrees-of-freedom),

$$K'_{mn} = \begin{cases} N, & m = n \\ 0, & m \neq n \end{cases} \qquad m = 0,1,2,\ldots,N-1; \ n = 0,1,2,3,\ldots,N-1 \qquad . \qquad (4.6)$$

Define an integer $\ell = m-n$, then from Eqns. (4.5) and (4.6)

$$\sum_{i=1}^{N} e^{j\ell\psi_i} = 0, \quad \begin{array}{c} -(N-1) \leq \ell \leq N - 1 \\ \ell \neq 0 \end{array} \qquad . \qquad (4.7)$$

It is desired to find the N values of ψ_i such that Eq. (4.7) is satisfied for all values of ℓ.

Equation (4.7) represents N phasors whose sum is zero. This suggests a uniform distribution of phasors between 0 and $2\pi\ell$, that is, assume a solution of the form

$$\psi_i = i \frac{2\pi}{N} \qquad (4.8)$$

where i is an integer ($i = 1,2,3,\ldots,N$) or ($i = 0,1,2,3,\ldots,N-1$).

21

Substituting Eq. (4.8) in the left-hand side of Eq. (4.7) yields

$$\sum_{i=1}^{N} e^{j(i\frac{2\pi}{N}\ell)} = T \quad . \tag{4.9}$$

It must be shown that the sum (denoted by T) is zero, for $\psi_i = i\, 2\pi/N$ to be a solution of Eq. (4.7).

If we set $\psi = 2\pi\ell/N$, the left-hand side of Eq. (4.9) is recognized as the array factor (AF) of a uniform linear array, which may be summed as follows:

$$AF = \left| \sum_{i=1}^{N} e^{ji\psi} \right| = \frac{\sin(N\psi/2)}{\sin \psi/2} = \left| \sum_{i=0}^{N-1} e^{ji\psi} \right| \tag{4.10}$$

so that

$$|T| = \frac{\sin(N\psi/2)}{\sin \psi/2} = \frac{\sin(N(\frac{2\pi\ell}{N})/2)}{\sin(2\pi\ell/2N)} = \frac{\sin \pi\ell}{\sin(\frac{\pi\ell}{N})} = 0 \quad . \tag{4.11}$$

$$-(N-1) \leq \ell \leq N-1$$

$$\ell \neq 0.$$

Since $|T| = 0$, $\psi_i = (i\,(2\pi/N))$ is a solution of Eq. (4.7) for all values of ℓ. We may now solve for θ_i

$$\psi_i = 2\pi d/\lambda \, \sin \theta_i = i\,\frac{2\pi}{N} \quad . \tag{4.12}$$

22

The orthogonal interference source angles are, thus,

$$\theta_i = \sin^{-1}(\frac{i}{N}\frac{\lambda}{d}) \qquad (4.13)$$

where $\quad -\frac{d}{\lambda} \leq \frac{i}{N} \leq \frac{d}{\lambda} \quad .$

The ratio i/N has been restricted to the range $-d/\lambda$ to d/λ so that θ_i is real. In the next section, Eq. (4.13) is used to find the configuration of eight orthogonal interference sources for an eight-element linear array.

C. Example: Eight-Element Linear Array

Consider an eight-element linear array of isotropic point sources with one-half wavelength interelement spacing, as shown in Figure 4.2. The angular locations of eight orthogonal sources are computed from Eq. (4.13). Using $d/\lambda = 1/2$ and $N = 8$, then

$$-N\frac{d}{\lambda} \leq i \leq N\frac{d}{\lambda} \qquad (4.14)$$

or

$$-3 \leq i \leq 4 \qquad (4.15)$$

are the possible values. (Note: $i = -4$ is excluded because it is the same as $i = +4$, that is,

23
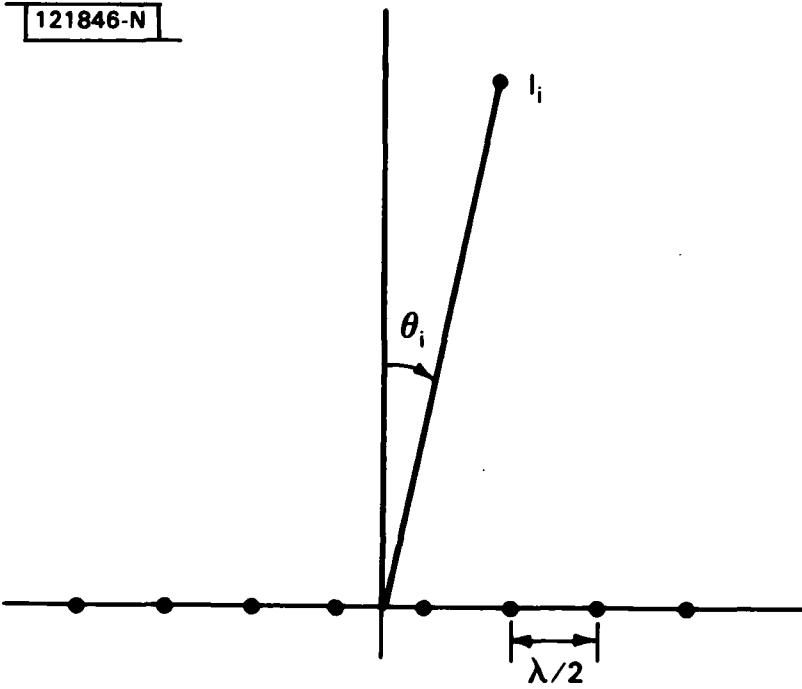
Figure 4.2 8-element linear array of one-half wavelength spaced isotropic elements with interference source at θ_i.

24

$$e^{-j4\frac{2\pi}{8}} = e^{j4\frac{2\pi}{8}} = e^{j\pi} .$$

Substituting this result in Eq. (4.13) yields

$$\theta_i = \sin^{-1}\left(\frac{1}{4}\right) \quad i = 0, \pm 1, \pm 2, \pm 3, +4 \quad . \tag{4.16}$$

To consume eight degrees of freedom, the interference position angles are computed to be $(0°, \pm 14.48°, \pm 30°, \pm 48.59°, +90°)$. The significance of these angles is clearly shown by the array factor (Eq. (4.10) with N = 8)) given in Figure 4.3. One source is placed at each null position, and one is positioned at the peak of the main beam. The above interference configuration was used as input data to an Applebaum-Howells adaptive nulling computer program. The quiescent receiving state is assumed here to be uniform coverage, that is, only a single array element is "on". Since the array elements are isotropic the quiescent directivity in the source directions is 0 dBi. The adapted results are given in Table 4.1. The eigenvalue spread is approximately 0 dB. The adapted directivity in the source directions is approximately 0 dBi, hence no pattern nulls have been formed. This verifies that the sources are orthogonal and that they consume eight degrees-of-freedom.

D. Discussion

The worst-case configuration of N interference sources, to completely consume N degrees-of-freedom of an N-element adaptive nulling linear array antenna of equally-spaced isotropic point sources, is an arc with source

25
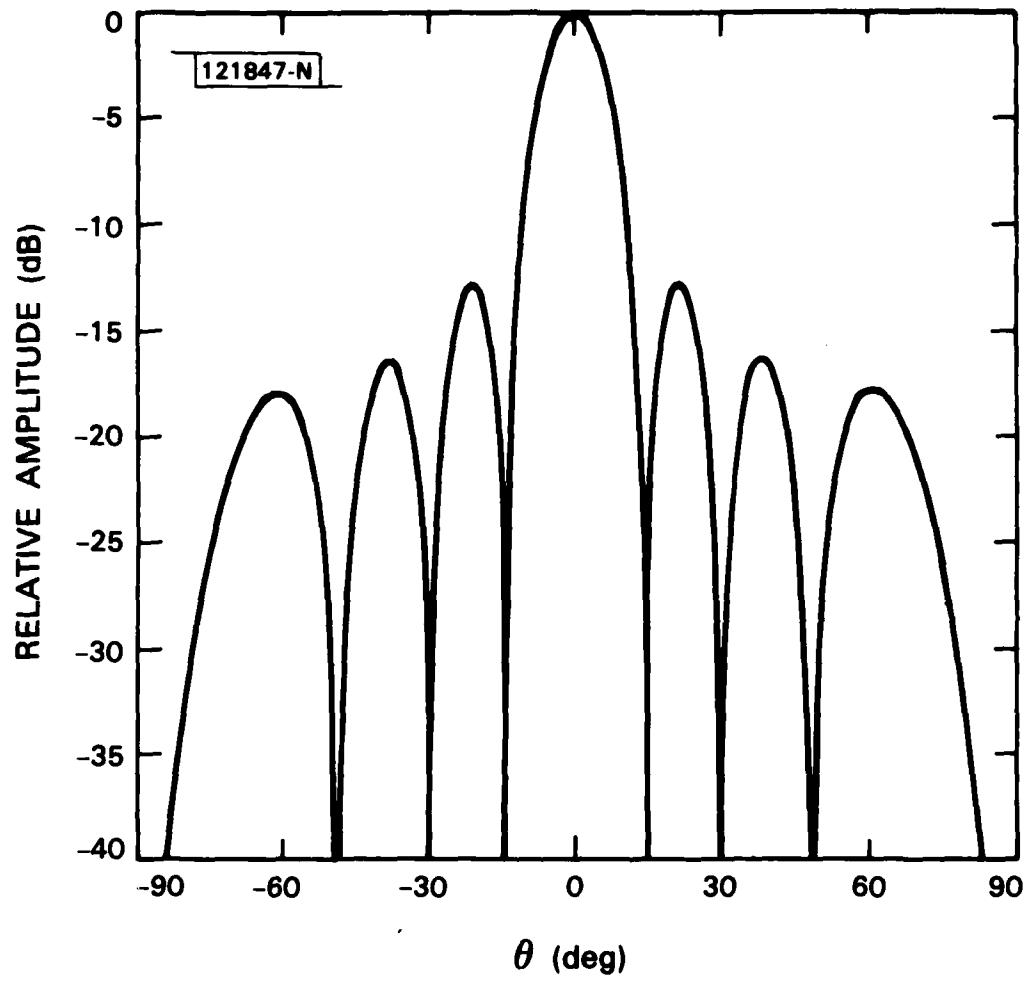
Figure 4.3 Array factor for 8-element linear array with one-half wavelength spacing.

TABLE 4.1

ADAPTED ANTENNA RESPONSE FOR AN 8-ELEMENT UNIFORM
LINEAR ARRAY WITH ONE-HALF WAVELENGTH SPACING

Eigenvalues

i	λ_i (dB)
1	22.8
2	22.8
3	22.8
4	22.8
5	22.8
6	22.8
7	22.8
8	22.7

ANTENNA DIRECTIVITY D_j AT INTERFERENCE SOURCE POSITION (θ_j, ϕ_j)

j	θ_j (degrees)	ϕ_j	D_j(Quiesent) dBi	D_j(Adapted) dBi
1	-48.59	0.0	0.0	0.04
2	-30.00	0.0	0.0	0.00
3	-14.48	0.0	0.0	-0.02
4	0.0	0.0	0.0	-0.02
5	14.48	0.0	0.0	-0.02
6	30.00	0.0	0.0	-0.02
7	48.59	0.0	0.0	0.00
8	90.00	0.0	0.0	0.04

spacing that is equal to the angular peak-to null spacing of the array factor for uniform amplitude. These interference sources are referred to as "orthogonal" because their locations are determined by requiring that the off-diagonal elements of the interference covariance matrix equal to zero. The orthogonal interference source position angles are computed from Eq. (4.13). This equation was used to find eight orthogonal interference sources for an eight-element array linear array. No adaptive nulling was possible for this arrangement of sources.

In the next section, a figure of merit is derived which can be used to determine the worst-case interference source configurations for an adaptive nulling array antenna with arbitrary element positions.

V. DERIVATION OF A FIGURE OF MERIT TO DETERMINE THE BEST APPROXIMATION TO A UNITARY INTERFERENCE SIGNAL MATRIX

A. Introduction

In Section III, it was shown that the optimum configuration of N equal-power interference sources, to consume N degrees of freedom of an N-channel adaptive nulling array antenna, produces a unitary signal matrix. The covariance matrix of the optimum signal matrix, normalized to the incident interference power, is diagonal. The diagonal elements are identical, and they are proportional to the incident interference power. While it is clear that a unitary signal matrix is optimum, the optimum interference source locations cannot be analytically computed for some arrays. This is shown by examining the covariance matrix of the N interference sources for an arbitrary array.

Let R_{mn} be the mn^{th} element of the interference covariance matrix without quiescent receiver noise (i.e., $\underline{\underline{SS}}^\dagger$). For interference sources to be "orthogonal", the elements of the covariance matrix must have the following property[4],

$$R_{mn} = \begin{cases} \displaystyle\sum_{i=1}^{N} P_i & m=n \\ \\ 0 & m \neq n \end{cases} \qquad \begin{array}{l} m=1,2,\ldots,N \\ n=1,2,\ldots,N \end{array} \qquad (5.1)$$

Using Eq. (3.1) and (3.2), the mn^{th} phase term in the covariance matrix $\underline{\underline{SS}}^\dagger$, containing the unknown interference position angles (θ_i, ϕ_i), is of the form

$$R_{mn} = \sum_{i=1}^{N} e^{j(\psi_{i,m} - \psi_{i,n})} \qquad (5.2)$$

29

where
$$\psi_{i,m} = \pi D/\lambda \, \sin \theta_i \left(x_m \cos \phi_i + y_m \sin \phi_i \right) \qquad (5.3)$$

$$m = 1, 2, \ldots, N$$

$$n = 1, 2, \ldots, N$$

$$\psi_{i,n} = \pi D/\lambda \, \sin\theta_i \left(x_n \cos\phi_i + y_n \sin\phi_i \right) \qquad (5.4)$$

If the orthogonality conditions of Eq. (5.1) are imposed on the mn^{th} terms of Eq. (5.2), together with the Hermitian symmetry of $\underline{R}$ (i.e., $\underline{R} = \underline{R}^{\dagger}$), then there are $N(N-1)/2$ complex equations with $2N$ unknowns (θ_i, ϕ_i), $i = 1, 2, \ldots, N$. For periodic arrays, the $N(N-1)/2$ complex equations usually reduce to $2N$ real equations. The $2N$ unknowns then can be analytically determined. However, for certain arrays (random, thinned, or irregular), there are generally more than $2N$ real equations and the system is overdetermined. There is no exact solution then, because the solution of a subset of $2N$ real equations will not (in general) satisfy the remaining equations.

From the above result, it is apparent that analytical solutions for orthogonal interference source locations for many arrays are not readily obtainable. It is the subject of this section to derive a figure of merit which can be used to select, by computer search, the best approximation to a unitary matrix. (This is equivalent to finding the best fit to the orthogonality conditions (Eq. 5.1)). The figure of merit is shown to be equal to the sum of the square roots of the eigenvalues computed from the interference covariance matrix. For a given set of interference source configurations, the one which consumes the most degrees of freedom maximizes the figure of merit. This criterion is applied in Section V.C to the case of two sources in the field of view of an N-element array.

30

B. Derivation of a Figure of Merit to Maximize Consumption of Degrees of
    Freedom

Let $\underline{S}$ be the matrix that is generated by the incident signals from N
sources, as given by Eq. (3.1). It is desired to minimize the difference
between $\underline{S}$ and a desirable unitary matrix $\underline{A}$. As discussed previously, a
unitary signal matrix implies that the interference sources are "orthogonal",
that is, N degrees of freedom are consumed. If $\underline{S}$ and $\underline{A}$ are approximately
equal, this represents a nearly orthogonal configuration of interference
sources. The following minimization is the same as that given in Appendix I
in a report by W. C. Cummings on multiple beam forming networks[6]. It is
repeated here to make the connection to orthogonal interference sources as
well.

First, define the difference between the given signal matrix and the
optimum unitary matrix as

$$\underline{D} = \underline{S} - \underline{A} \tag{5.5}$$

(For convenience, the double underbar (for square matrices) will now be
dropped.) As will be shown, the square matrix A can be computed from the
eigenvector matrices of $SS^\dagger$ and $S^\dagger S$. One way to minimize D is to minimize the
sum of the squares of the magnitudes of each term in D. That is,

$$||D||^2 = \sum_{ij} |D_{ij}|^2 = \text{minimum} \tag{5.6}$$

31

or

$$||S-A||^2 = \text{minimum} \qquad (5.7)$$

By the singular value decomposition theorem[7] any matrix, such as S, can be decomposed as

$$S = V \Sigma T^\dagger \qquad (5.8)$$

where

$$\Sigma = \text{diag}(\sigma_i) \qquad i=1,2,\ldots,N \qquad (5.9)$$

$\sigma_i = \sqrt{\lambda_i}$ are the positive square roots of the eigenvalues of $SS^\dagger$ (and of $S^\dagger S$, equivalently), and are referred to as the singular values of S

V   is the unitary eigenvector matrix of $S\,S^\dagger$

T   is the unitary eigenvector matrix of $S^\dagger S$.

Substituting Eq. (5.8) in Eq. (5.7) yields

$$||V\Sigma T^\dagger - A||^2 = \text{minimum} \qquad (5.10)$$

32

Next, use the unitary matrix property that[7]

$$||M||^2 = ||U_1 M||^2 = ||MU_2||^2 = ||U_1 M U_2||^2 \qquad (5.11)$$

where U_1 and U_2 are unitary matrices; and

M is an NxN matrix.

Pre-multiplying by $V^\dagger$ and post-multiplying by T, Eq. (5.10) becomes

$$||V\Sigma T^\dagger - A||^2 = ||V^\dagger V\Sigma T^\dagger T - V^\dagger AT||^2 \qquad . . \qquad (5.12)$$

Since V and T are unitary, that is, $V^\dagger V = I = T^\dagger T$,

$$V^\dagger V\Sigma T^\dagger T = I\Sigma I = \Sigma \qquad (5.13)$$

Substituting this in Eq. (5.12) yields

$$||V\Sigma T^\dagger - A||^2 = ||\Sigma - V^\dagger AT||^2 \qquad . \qquad (5.14)$$

Let

$$P = V^\dagger AT \qquad (5.15)$$

33

so that Eq. (5.14) is written as

$$||V\Sigma T^\dagger - A||^2 = ||\Sigma - P||^2 = \text{minimum} \qquad . \qquad (5.16)$$

Now P is unitary since

$$P^\dagger P = T^\dagger A^\dagger VV^\dagger AT = T^\dagger A^\dagger AT = T^\dagger T = 1 \qquad . \qquad (5.17)$$

Hence

$$\sum_{j=1}^{N} |P_{ij}|^2 = 1 \qquad , \qquad (5.18)$$

that is, the sum of the absolute values of the elements along any column is unity. Also, because P is unitary, the diagonal elements are constrained by

$$|P_{ii}| \leq 1 \qquad . \qquad (5.19)$$

Another useful property of the unitary matrix is that

$$\sum_{i=1}^{N} \sum_{j=1}^{N} |P_{ij}|^2 = N \qquad . \qquad (5.20)$$

Since Σ is diagonal it is logical that, to minimize $||\Sigma - P||^2$, the optimum P should also be diagonal (this is the only way that the off-diagonal elements of $(\Sigma - P)$ can be equal to zero). Therefore, by Eqns. (5.18) and (5.19), P must

be the identity matrix. It must now be shown that of all unitary matrices, P=I is the closest unitary matrix to Σ.

To show this it is required to prove that

$$||\Sigma-I||^2 \leq ||\Sigma-U||^2 \qquad (5.21)$$

for all unitary matrices U. Performing the norm on the left and right-hand sides of the above inequality yields

$$||\Sigma-I||^2 = \sum_{i=1}^{N} (\sigma_i-1)^2 = \sum_{i=1}^{N} \sigma_i^2 - 2 \sum_{i=1}^{N} \sigma_i + N \qquad (5.22)$$

and

$$||\Sigma-U||^2 = \sum_{\substack{i=1 \\ i\neq j}}^{N} \sum_{j=1}^{N} |U_{ij}|^2 + \sum_{i=1}^{N} |\sigma_i-U_{ii}|^2$$

or

$$||\Sigma-U||^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} |U_{ij}|^2 + \sum_{i=1}^{N} \sigma_i^2 - \sum_{i=1}^{N} \sigma_i(U_{ii}+U_{ii}^*) \qquad (5.23)$$

where * denotes complex conjugate. Using Eq. (5.20), the double summation on $|U_{ij}|^2$ is equal to N, so that Eq. (5.23) reduces to

$$||\Sigma-U||^2 = N + \sum_{i=1}^{N} \sigma_i^2 - \sum_{i=1}^{N} \sigma_i(U_i + U_{ii}^*) \qquad . \qquad (5.24)$$

Substituting Eqns. (5.22) and (5.24) in Eq. (5.21) gives

$$\sum_{i=1}^{N} \sigma_i^2 + N - 2 \sum_{i=1}^{N} \sigma_i \le \sum_{i=1}^{N} \sigma_i^2 + N - \sum_{i=1}^{N} \sigma_i (U_{ii} + U_{ii}^*) \quad . \qquad (5.25)$$

Clearly, if the unitary matrix U is other than the identity matrix, then by Eqs. (5.18) and (5.19)

$$(U_{ii} + U_{ii}^*) < 2 \qquad , \qquad (5.26)$$

and the inequality of Eq. (5.25) (and Eq. (5.21)) is satisfied. Therefore, P=I is the optimum matrix which minimizes Eq. (5.16).

Using the left-hand side of Eq. (5.25), Eq. (5.16) can now be written as

$$||\Sigma - P||^2 = \sum_{i=1}^{N} \sigma_i^2 + N - 2 \sum_{i=1}^{N} \sigma_i = \text{minimum} \qquad . \qquad (5.27)$$

In Eq. (5.27) the summation of σ_i^2, N and $||\Sigma - P||^2$ are all positive, hence $||\Sigma - P||^2$ is minimized when

$$F = \sum_{i=1}^{N} \sigma_i = \sum_{i=1}^{N} \sqrt{\lambda_i} = \text{maximum} \qquad (5.28)$$

Equation (5.28) is the desired figure of merit (denoted by F), that is, when the interference sources are located so that the sum of the square roots of the eigenvalues computed from the covariance matrix $SS^\dagger$ is maximized, the signal matrix S is the best approximation to the unitary matrix A. The

optimum unitary matrix A can be found by substituting P=I in Eq. (5.15), that
is,

$$P = V^{\dagger} A T = I \tag{5.29}$$

or
$$V P T^{\dagger} = V V^{\dagger} A T T^{\dagger} = V I T^{\dagger} \quad . \tag{5.30}$$

Since $V V^{\dagger} = T T^{\dagger} = I$, then

$$A = V T^{\dagger} \tag{5.31}$$

is the optimum unitary matrix which minimizes $||S-A||^2$. Note: for any given
signal matrix S, a unitary matrix A can be computed by Eq. (5.31).

Finally, denote F_j as the figure of merit for the jth configuration of
interference sources, from a given set of J configurations. Then the optimum
interference source configuration (to consume the most degrees of freedom) is
chosen according to

$$F_{opt} = \max(F_j) \qquad j=1,2,\ldots,J \tag{5.32}$$

where

$$F_j = \sum_{i=1}^{N} \sqrt{\lambda_{ij}} \quad .$$

Equation (5.32) is the basis for choosing one configuration of interference sources over another. An example of this is given in the next section.

C. Example: Selection of Optimum Configuration of Two Interference Sources

In this section an example is given which demonstrates that the figure of merit, given by Eq. (5.32), can be used to determine the worst-case[*] configuration of two interference sources.

Consider an arbitrary N-element array of isotropic point sources, with equal power (P) incident from two interference sources with two sets of spacings. In the first set, the two sources are angularly separated (approximately one half-power beamwidth) such that there are two equal eigenvalues $\lambda_1 = \lambda_2 = \lambda$ associated with the covariance matrix. Thus, two degrees of freedom are consumed[1]. It is assumed here that λ is much larger than the quiescent noise level ($\lambda_q = 1$). The (N-2) remaining eigenvalues are equal to unity. From Eq. (5.28) the figure of merit is

$$F_1 = \sum_{i=1}^{N} \sqrt{\lambda_i} = \sqrt{\lambda_1} + \sqrt{\lambda_2} + (N-2)$$

or

$$F_1 = 2\sqrt{\lambda} + (N-2) \tag{5.33}$$

---

[*]The words "optimum configuration" and "worst-case configuration" are used somewhat interchangeably in this report. A situation is "optimum" or "worst-case" depending on the point of view.

Next, assume that the two interference sources are located at the same angular position. In this case, there is only one eigenvalue and it is 3 dB larger than the eigenvalue for a single interference source[1]. That is,

$$\lambda_1 = 2\lambda, \; \lambda_2 = 1, \; \lambda_3 = 1, \; \ldots, \; \lambda_N = 1$$

and one degree of freedom is consumed. For this case, the figure of merit is computed to be

$$F_2 = \sum_{i=1}^{N} \sqrt{\lambda_i} = \sqrt{2} \, \sqrt{\lambda} + (N-1) \quad . \tag{5.34}$$

By Eq. (5.32), since F_1 is greater than F_2, configuration #1 has more effect as an interference source than configuration #2.

D. Discussion

A figure of merit (Eq. (5.28)) was derived which can be used to determine which matrix, of a collection of interference signal matrices, is the best approximation to a unitary matrix. For a given number of interference source configurations, the worst-case is chosen corresponding to the configuration which maximizes the sum of the square roots of the eigenvalues computed from the interference covariance matrix (Eq. (5.32)). This is the same as maximizing the consumption of the antenna degrees of freedom. The reason for this is that this optimum is closest to being a unitary matrix and when a unitary interference signal matrix is achieved, N degrees of freedom are completely consumed (there are N identical eigenvalues).

39

It is important to note that the figure of merit is independent of the antenna quiescent radiation pattern. This is because the covariance matrix is formed prior to array element weighting or beam formation.

A simple example was given which demonstrates the ability of the figure of merit to identify a two-interference source configuration that maximizes the consumption of antenna degrees of freedom. The example was an N-element array with two equal-power interference sources (first located with one-half beamwidth spacing, and then located at the same position). It was shown that the figure of merit can be used to choose the configuration that consumes two degrees of freedom. The next section discusses a gradient-search technique, which implements the figure of merit to find worst-case interference source configurations for arbitrary antenna arrays.

VI. A GRADIENT SEARCH TECHNIQUE TO MAXIMIZE CONSUMPTION OF PHASED ARRAY
     ANTENNA DEGREES OF FREEDOM

A. Introduction

In the previous sections, a mathematical formulation was developed which
describes the conditions necessary for the consumption of phased array antenna
degrees of freedom. Complete consumption of N degrees of freedom of an
N-element phased array antenna occurs when the N eigenvalues computed from the
interference covariance matrix are equal and large compared to the quiescent
receiver noise level. For N equal eigenvalues, the covariance matrix is
diagonal and the interference signal matrix is unitary. Since the columns of
the signal matrix are orthogonal, the interference sources are referred to as
being "orthogonal". It was shown that when N interference sources are
geometrically arranged such that N large equal eigenvalues are produced, the
antenna cannot provide any useful adapted pattern nulling.

For some periodic arrays, exact solutions for orthogonal interference
sources can be found. However, for some arrays such as random, thinned, or
irregular arrays, exact solutions do not appear to exist. In this case
N degrees of freedom cannot be completely consumed by N interference
sources. That is, some antenna pattern nulling is possible. Since a unitary
interference signal matrix completely consumes N degrees of freedom, the
interference signal matrix must be the closest approximation to a unitary
matrix in order to maximize consumption of degrees of freedom. The closest
approximation to a unitary interference signal matrix for an arbitrary array
can be realized in the following manner:

41

From Eq. (3.1), let the received signal at the kth array element be given by

$$S_{kij} = \sqrt{P_i} \; e^{j\pi D/\lambda \sin\theta_{ij}(x_k \cos \phi_{ij} + y_k \sin \phi_{ij})} \tag{6.1}$$

where (θ_{ij}, ϕ_{ij}) are standard spherical coordinate angles for the ith interference source with power P_i measured at the antenna; the subscript j being the jth configuration of interference sources. (Note: the imaginary exponent $j = \sqrt{-1}$ should not be confused with the integer subscript $j=1,2,\ldots$.). The element positions (x_k, y_k) can be arbitrarily chosen.

The covariance matrix for narrowband interference is expressed as (from Eq. (3.2)

$$\underset{=}{R}_j = \underset{=}{S}_j \underset{=}{S}_j^{\dagger} + \underset{=}{I} \tag{6.2}$$

To maximize consumption of antenna degrees of freedom the difference between the signal matrix $\underset{=}{S}_j$ and its associated unitary matrix must be minimized. A derivation was given in Section V for a figure of merit which minimizes this difference.

The figure of merit for the jth interference source configuration is given by

$$F_j = \sum_{i=1}^{N} \sqrt{\lambda_{ij}} \tag{6.3}$$

42

where λ_{ij} is the ith eigenvalue computed from the interference covariance matrix for the jth interference source configuration.

The optimum interference source configuration (from a given set of J source configurations) occurs when F_j is maximized, that is,

$$F_{opt} = \max(F_j) \qquad j=1,2,\ldots,J \qquad . \tag{6.4}$$

The interference source configuration for which F_{opt} occurs yields the closest approximation of the signal matrix $\underline{S}$ to a unitary matrix.

In the next section, a gradient search technique is introduced which implements the figure of merit. In Section VI.C, the gradient search is applied to two basic planar arrays.

B. Gradient Search Technique

Assume that N interference sources are distributed in the antenna field of view as shown in Fig. 6.1. The ith interference source from the jth source configuration has position coordinates (U_{ij}, V_{ij}) where

$$U_{ij} = \pi D/\lambda \sin\theta_{ij} \cos\phi_{ij}$$

$$\tag{6.5}$$

$$V_{ij} = \pi D/\lambda \sin\theta_{ij} \sin\phi_{ij}$$

and (θ_{ij}, ϕ_{ij}) are standard spherical coordinates. It is desired to find the configuration of N interference sources such that the figure of merit, given

43

FIELD-OF-VIEW

ith SOURCE

(θ_{ij}, ϕ_{ij})

$$U = \pi \ \frac{D}{\lambda} \ \sin \theta \cos \phi$$

$$V = \pi \ \frac{D}{\lambda} \ \sin \theta \cos \phi$$
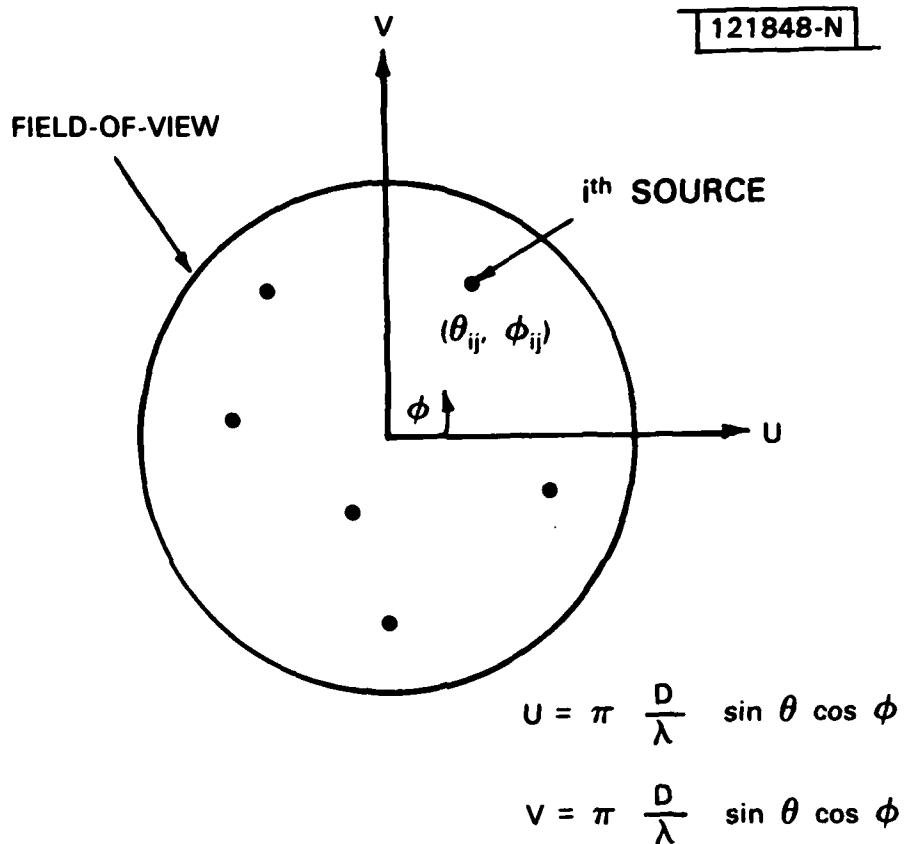
Figure 6.1 Distribution of N interference sources.

44

oy Eq. (6.3), is maximized. Assuming an initial configuration of sources, the sources are moved until the optimum figure of merit is achieved. From Eqs. (6.1), (6.2), (6.3), and (6.5) observe that

$$\lambda_{ij} = \lambda_{ij}((U_{1j}, V_{1j}), (U_{2j}, V_{2j}), \ldots, (U_{Nj}, V_{Nj})) \qquad , \qquad (6.6)$$

which means that each eigenvalue is a function of the positions of N sources. It is desired to find the collective search-directions for the N sources such that the figure of merit increases most rapidly. That is, select directions such that the directional derivative is maximized at $(\overline{U}_j, \overline{V}_j)$[8].

The directional derivative (denoted by D( )) of the figure of merit is given by

$$D(F_j) = \sum_{i=1}^{N} (\frac{\partial F_j}{\partial U_{ij}} r_{uij} + \frac{\partial F_j}{\partial V_{ij}} r_{vij}) \qquad (6.7)$$

where ∂ means partial derivative,

r_{uij}, r_{vij} are the (U,V) directions for which F_j is increasing most rapidly.

The directions r_{uij}, r_{vij} are constrained by (for convenience)

$$\sum_{i=1}^{N} (r_{uij}^2 + r_{vij}^2) = 1 . \qquad (6.8)$$

It is desired to maximize $D(F_j)$ subject to Eq. (6.8). Using Lagrange multipliers[9] construct the Lagrangian function

$$L_j = \sum_{i=1}^{N} \left(\frac{\partial F_j}{\partial U_{ij}} r_{uij} + \frac{\partial F_j}{\partial V_{ij}} r_{vij} \right) + \gamma \left(1 - \sum_{i=1}^{N} (r_{uij}^2 + r_{vij}^2) \right) \qquad (6.9)$$

where γ is a constant to be determined. The requirement that L_j be an extremum implies

$$\frac{\partial L_j}{\partial r_{unj}} = \frac{\partial F_j}{\partial U_{nj}} - 2\gamma r_{unj} = 0 \qquad n=1,2,\ldots,N \qquad (6.10a)$$

$$\frac{\partial L_j}{\partial r_{vnj}} = \frac{\partial F_j}{\partial V_{nj}} - 2\gamma r_{vnj} = 0 \qquad n=1,2,\ldots,N \qquad (6.10b)$$

or

$$r_{unj} = \frac{1}{2\gamma} \frac{\partial F_j}{\partial U_{nj}} \qquad (6.11a)$$

$$r_{vnj} = \frac{1}{2\gamma} \frac{\partial F_j}{\partial V_{nj}} \qquad . \qquad (6.11b)$$

Squaring Eqns. (6.11a) and (6.11b) and invoking Eq. (6.8) yields

$$\sum_{n=1}^{N} (r_{unj}^2 + r_{vnj}^2) = 1 = \frac{1}{4\gamma^2} \sum_{n=1}^{N} \left(\left(\frac{\partial F_j}{\partial U_{nj}}\right)^2 + \left(\frac{\partial F_j}{\partial V_{nj}}\right)^2 \right) \qquad (6.12)$$

thus,

$$\gamma = \pm \frac{1}{2} \sqrt{\sum_{n=1}^{N} \left(\left(\frac{\partial F_j}{\partial U_{nj}}\right)^2 + \left(\frac{\partial F_j}{\partial V_{nj}}\right)^2 \right)} \qquad (6.13)$$

Substituting this result in Eqs. (6.11a) and (6.11b) gives

$$r_{unj} = \frac{\frac{\partial F_j}{\partial U_{nj}}}{\sqrt{\sum_{n=1}^{N} \left(\left(\frac{\partial F_j}{\partial U_{nj}}\right)^2 + \left(\frac{\partial F_j}{\partial V_{nj}}\right)^2 \right)}} \qquad (6.14a)$$

$$r_{vnj} = \frac{\frac{\partial F_j}{\partial V_{nj}}}{\sqrt{\sum_{n=1}^{N} \left(\left(\frac{\partial F_j}{\partial U_{nj}}\right)^2 + \left(\frac{\partial F_j}{\partial V_{nj}}\right)^2 \right)}} \cdot \qquad (6.14b)$$

The positive sign was chosen corresponding to the direction of maximum function increase. The partial derivatives

$$\frac{\partial F_j}{\partial U_{nj}} \ , \ \frac{\partial F_j}{\partial V_{nj}} \ ; \ n=1,2,\ldots,N$$

represent the gradient directions for maximum function increase.

Since Eq. (6.3) cannot, in general, be expressed in a functional form, the partial derivatives must be numerically computed. Computation of the partial derivatives can be avoided, however, by using the following approach:

Write

$$\frac{\partial F_j}{\partial U_{nj}} = \frac{\Delta F_{unj}}{2\Delta U_{nj}} \qquad (6.15a)$$

$$\frac{\partial F_j}{\partial V_{nj}} = \frac{\Delta F_{vnj}}{2\Delta V_{nj}} \qquad (6.15b)$$

47

where as shown in Fig. 6.2

$$\Delta F_{uuj} = F_j(U_{nj} + \Delta U_{nj}; V_{nj}) - F_j(U_{nj} - \Delta U_{nj}; V_{nj}) \tag{6.16a}$$

$$\Delta F_{vnj} = F_j(U_{nj}; V_{nj} + \Delta V_{nj}) - F_j(U_{nj}; V_{nj} - \Delta V_{nj}) \tag{6.16b}$$

ΔU_{nj} and ΔV_{nj} are assumed to be small increments.

The problem is simplified (and the search is unbiased) if the increments ΔU_{nj} and ΔV_{nj} are taken to be equal, that is,

$$\Delta U_{nj} = \Delta V_{nj} = \Delta U_n = \Delta V_n \tag{6.17}$$

Substituting Eqs. (6.15) and (6.17) in Eqs. (6.14a) and (6.14b) yields

$$r_{unj} = \frac{\Delta F_{unj}}{\sqrt{\sum_{n=1}^{N} ((\Delta F_{unj})^2 + (\Delta F_{vnj})^2)}} \tag{6.18a}$$

$$r_{vuj} = \frac{\Delta F_{vnj}}{\sqrt{\sum_{n=1}^{N} ((\Delta F_{unj})^2 + (\Delta F_{vnj})^2)}} \tag{6.18b}$$

Equations (6.18a) and (6.18b) are used to compute the new positions of the $(j+1)^{th}$ configuration according to

$$U_{n(j+1)} = U_{nj} + \Delta U_n \, r_{unj} \tag{6.19a}$$

48
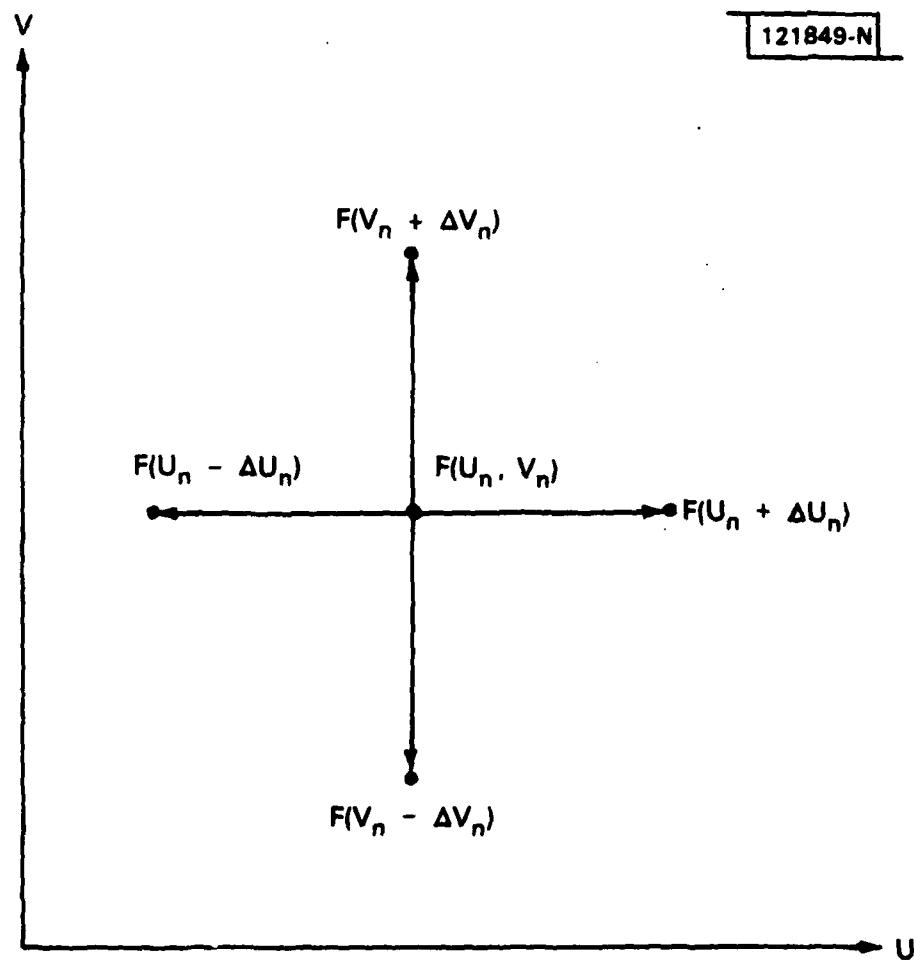
$$\Delta F_{v,n} = F(V_n + \Delta V_n) - F(V_n - \Delta V_n)$$

$$\Delta F_{u,n} = F(U_n + \Delta U_n) - F(U_n - \Delta U_n)$$

Figure 6.2 Figure of merit increments for optimum search directions.

49

$$V_{n(j+1)} = V_{nj} + \Delta V_n r_{vnj} \qquad\qquad (6.19b)$$

In practice, one interference source remains fixed in position throughout the search, for reference purposes. Additionally, a random noise matrix, typically 30 dB below the incident interference power, is added to the covariance matrix to avoid testing for either a minimum or saddle point of the figure of merit[10].

C. Application

A computer program implementing the above gradient search technique is used in this section to determine the worst-case interference source locations for two basic arrays. (In general, the array elements can have arbitrary positions given in (x,y,z) rectangular coordinates.) Two arrays that are useful in testing the gradient search are shown in Fig. 6.3. Figure 6.3a shows a seven-element array arranged in a regular hexagonal (equilateral triangular) grid for which an exact solution exists for the locations of seven interference sources to completely consume seven degrees of freedom. These positions can be determined by using Eqs. (6.1) and (6.2) and enforcing

$$R_{mn} = 0 \qquad m \neq n \qquad m=1,2,\ldots,N \quad n=1,2,\ldots,N \qquad (6.20)$$

that is, the covariance matrix is constrained to be diagonal. It can be shown analytically that the optimum configuration of interference sources (to obtain a 0 dB eigenvalue spread) is a regular hexagon, rotated with respect to the hexagonal array. (This derivation will not be given here.) Next, Figure 6.3b

121850-N

(a) HEXAGONAL

D = 65.7 λ

(b) RING
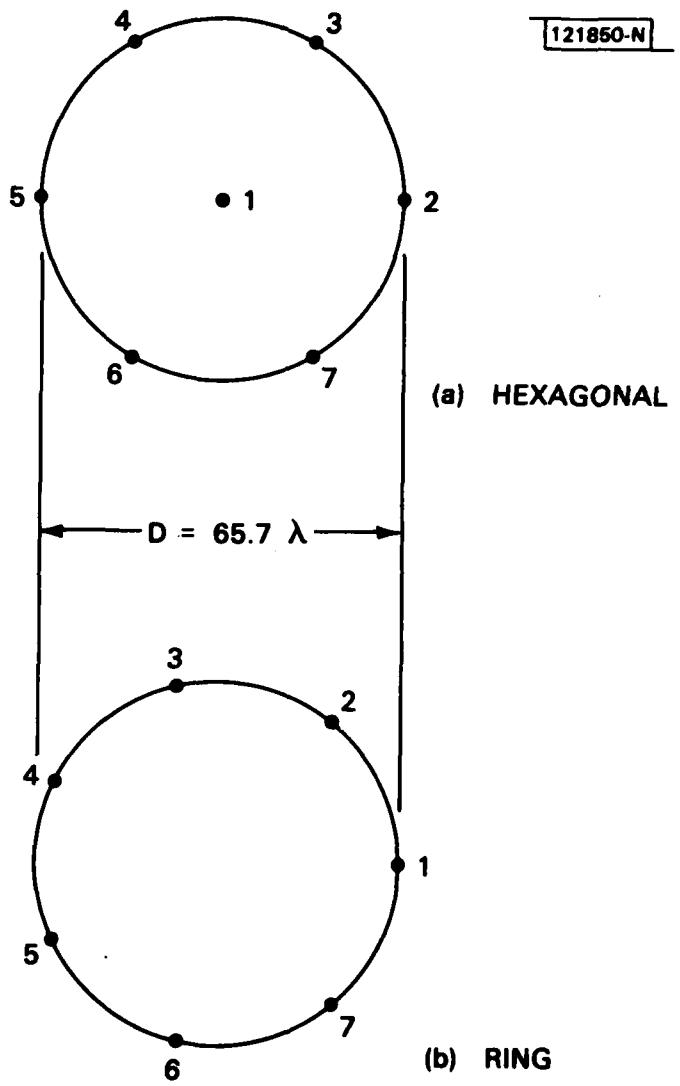
Figure 6.3 7-element array configurations.

51

shows a seven-element uniform circular ring array for which no exact solution is known to exist for seven interference sources to completely consume seven degrees of freedom. That is, Eq. (6.20) cannot be satisfied exactly. Since the interference source configuration which would maximize consumption of degrees of freedom for this antenna is unknown, a numerical solution is necessary.

Both the hexagonal array and ring array were chosen with an aperture diameter equal to 65.7λ. (These two arrays are, thus, highly thinned.) The array elements are assumed to be isotropic for the gradient search. However, for adapted antenna response computations, elements with a half-power beamwidth $\cong 18°$ pointed perpendicular to the plane of the array were used. At interference source locations close to boresight, the element directivity is approximately equal to 20 dBi. The quiescent mode of operation is assumed to be uniform coverage so that, when there is no interference, only one array element is "on". Note: the gradient search is independent of the quiescent mode of operation because the figure of merit is independent of array element excitation.

The initial interference source configuration is chosen to be located essentially at a single point within the field of view. That is, the starting configuration prior to the gradient search consumes only one degree of freedom. The seven sources appear as a single source with seven times the power of one of the seven sources. It is implied here that the starting configuration is unbiased. For convenience, in the two gradient searches that follow, actually only one interference source is fixed at boresight

(reference) while the remaining six sources are uniformly spaced on a ring (centered at boresight) whose radius is equal to 0.01 HPBW.

A gradient search for the 7-element hexagonal array (Fig. 6.3a) is given in Fig. 6.4. A typical source trajectory is shown with an arrow. First, the source moves radially outward from the reference source. After the maximum radius is achieved[*], the source rotates clockwise to its final position. (By symmetry, the source could also have moved counter-clockwise). The computed eigenvalue spread for the final hexagonal source configuration is 0.1 dB. The adapted antenna response for these sources is summarized in Table 6.1. The adapted directivity in each source direction is nearly 20 dBi, indicating that no adaptive nulls are formed. Thus, seven degrees of freedom are consumed for this array/source configuration, and there is no improvement in interference to receiver noise ratio after adaptation.

Next, a gradient search for the seven element uniform circular ring array (Fig. 6.3b) is shown in Fig. 6.5. A trajectory for one of the sources shows that the initial movement is purely radial. The final source configuration is U-shaped and produces a 6.4 dB eigenvalue spread. (By symmetry, the U-configuration can have other rotation angles but the shape is constant.) The source spacing is approximately equal to the peak-to-null spacing of the array factor. The adapted antenna response is summarized in Table 6.2. Since the eigenvalue spread was greater than 0 dB, some pattern nulling is possible. The improvement in interference-to-receiver noise level after

[*]At the maximum radius, the source spacing is approximately equal to the peak-to-null spacing of the array factor.

53

TABLE 6.1

ADAPTED ANTENNA RESPONSE FOR A 7-ELEMENT HEXAGONAL ARRAY
(Figure 6.4 source configuration)

Eigenvalues

i	λ_i (dB)
1	22.2
2	22.2
3	22.2
4	22.2
5	22.1
6	22.1
7	22.1

ANTENNA DIRECTIVITY AT INTERFERENCE SOURCE POSITION (θ_j, ϕ_j)

j	θ_j (degrees)	ϕ_j	D_j(Quiescent) dBi	D_j(Adapted) dBi
1	0.0	0.0	20.00	19.94
2	0.89	-10.75	20.00	20.03
3	0.89	49.03	20.00	19.99
4	0.89	108.92	20.00	19.94
5	0.89	169.35	20.00	19.97
6	0.89	-130.35	20.00	19.97
7	0.89	-70.45	20.00	19.98

TABLE 6.2

ADAPTED ANTENNA RESPONSE FOR A 7-ELEMENT UNIFORM RING ARRAY
(Figure 6.5 source configuration)

Eigenvalues

i	λ_i (dB)
1	24.3
2	24.2
3	24.2
4	22.0
5	20.2
6	19.5
7	17.9

ANTENNA DIRECTIVITY AT INTERFERENCE SOURCE POSITION (θ_j, ϕ_j)

j	θ_j	ϕ_j	D_j(Quiescent) dBi	D_j(Adapted) dBi
1	0.0	0.0	20.00	19.26
2	0.67	6.33	20.00	13.48
3	0.75	67.57	20.00	20.68
4	1.05	108.26	20.00	18.92
5	1.02	-160.48	20.00	11.03
6	0.73	-118.30	20.00	13.12
7	0.66	-56.21	20.00	21.07

nulling is, however, calculated to be only 1.76 dB. The directivity to several sources is somewhat reduced, but no deep nulls are formed. Thus, while a solution which results in a unitary signal matrix has not been found, a solution for effective consumption of degrees of freedom appears to have been reached.

D. Discussion

A gradient search technique is given which can be used to determine numerically the optimum geometrical locations of N interference sources to maximize consumption of degrees of freedom of an N-element phased array nulling antenna. The figure of merit, used to determine optimum source directions, is equal to the sum of the square roots of the eigenvalues computed from the interference covariance matrix.

Optimum solutions were given for two basic arrays; a seven-element hexagonal array and a seven-element uniform circular ring array. For the seven-element hexagonal array, an exact solution was found (0 dB eigenvalue spread). Seven interference sources on a rotated regular hexagon (Fig. 6.4) were shown to completely consume seven degrees of freedom (Table 6.1). For the seven-element uniform ring array, a 6.4 dB eigenvalue spread (Table 6.2) was achieved. Hence, an exact solution for this array geometry does not appear to exist. The worst-case interference source configuration in this case is U-shaped (Fig. 6.5). For either array, the interference source spacing is fundamentally related to the peak-to-null spacing of the array factor. Equivalently, the minimum spacing between two "optimum" sources is related to the antenna half-power beamwidth.

121851-N

"UV SPACE"

$\phi = 0°$
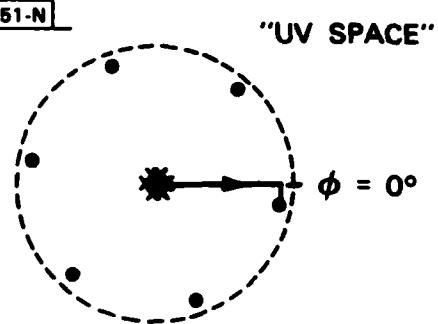
7 SOURCES

7 ELEMENTS
HEXAGONAL ARRAY

$D/\lambda = 65.7$

× STARTING CONFIGURATION
● FINAL CONFIGURATION
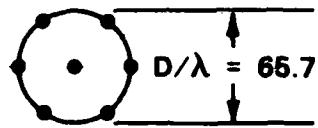
EIGENVALUE SPREAD = 0.18 dB

Figure 6.4   Gradient search for a 7-element hexagonal array.

57

121852-N

"UV SPACE"

$\phi = 0°$
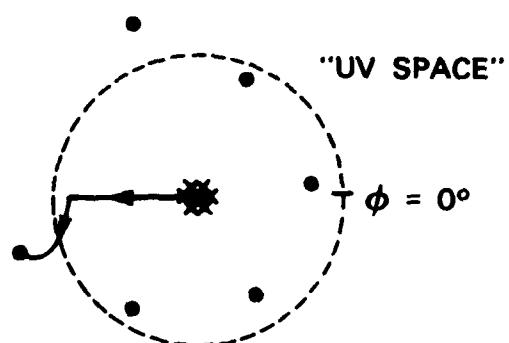
7 SOURCES

7 ELEMENTS
UNIFORMLY SPACED
ON A RING

$D/\lambda = 65.7$

× STARTING CONFIGURATION
● FINAL CONFIGURATION

EIGENVALUE SPREAD = 6.4 dB

Figure 6.5 Gradient search for a 7-element uniform ring array.

VII. CONCLUSIONS

A theory is developed for determining the locations of N-interference sources to maximize consumption of the degrees of freedom of an N-channel adaptive nulling phased array antenna. The worst-case arrangement of sources is determined by maximizing a figure of merit which is equal to the sum of the square roots of the eigenvalues computed from the interference covariance matrix. A gradient search technique is used to determine optimum source directions for an initial arrangement of sources in the antenna field of view. The initial source configuration is arbitrary, but for an unbiased solution, the sources are initially constrained to be spaced much less than the nulling antenna half-power beamwidth. That is, initially the sources consume only one degree of freedom.

Maximizing the figure-of-merit is equivalent to finding an incident-signal matrix which is the best approximation to a unitary matrix. Equivalently, the worst-case interference diagonalizes the interference covariance matrix. When the covariance matrix is diagonal, the sources may be referred to as "orthogonal". In this case, N-degrees of freedom are consumed, and the covariance matrix has N equal eigenvalues. When N-degrees of freedom are completely consumed by N sources, no adaptive nulling is possible.

For simple regular-spaced arrays, the concept of orthogonal interference sources can be used to find the worst-case interference geometry in a closed-form equation. This was done in Section IV for an eight-element equally-spaced linear array. However, for more complicated array geometries (such as random, thinned, or irregular arrays), the worst-case source configuration is

59

not mathematically tractable in a closed form. Indeed, a set of orthogonal sources probably does not exist, in general. A gradient search is appropriate in these cases to find the best approximation to a set of orthogonal sources.

In Section V, a figure of merit was derived which is used in the gradient search technique discussed in Section VI. The gradient search was applied to two different array geometries; a 7-element regular hexagonal array and a 7-element equally-spaced circular ring array. A numerical gradient search found an orthogonal configuration of seven sources, which completely consumed seven degrees of freedom for the hexagonal array. For the circular ring array, an orthogonal set of sources was not found; however, the worst-case solution severely reduced the amount of antenna pattern discrimination. The gradient search technique is valid for arbitrary array geometries. The element positions can be periodic, spatially tapered, thinned, irregular, random, planar, or non-planar.

Finally, only narrow bandwidth examples were considered here, so one interference source could consume no more than one degree of freedom. As is well known, broadband sources can contribute to more than one eigenvalue, thus complicating the simpler picture presented here. It is believed that the figure of merit presented will still give worst-case configurations in the broadband case, if the covariance matrix is properly calculated using Eq. (2.2).

APPENDIX A

In Eq. (3.5) it was stated that when N degrees of freedom are consumed, by N equal-power interference sources, the covariance matrix of an N element array has N identical eigenvalues such that

$$\lambda_1 = \lambda_2 = \ldots = \lambda = (PN+1) \tag{A.1}$$

where

P is the incident power of each interference source on each array

element.

This equation is derived in the following manner: Assume that the N interference sources are located at (θ_1,ϕ_1), (θ_2,ϕ_2), ..., (θ_N,ϕ_N). For convenience, express the interference signal matrix (Eq. 3.1) as

$$\underline{\underline{S}} = \sqrt{P} \; \underline{\underline{J}} \tag{A.2}$$

where

$$
J = \begin{bmatrix}
e^{ja_1} & e^{ja_2} & \cdots & e^{ja_N} \\
e^{jb_1} & e^{jb_2} & \cdots & e^{jb_N} \\
\cdot & \cdot & \cdot & \cdot \\
\\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
e^{jn_1} & e^{jn_2} & \cdot & e^{jn_N}
\end{bmatrix} \tag{A.3}
$$

where

$$a_i = \pi D/\lambda \, \sin \theta_i \, (x_1 \cos\phi_i + y_1 \sin\phi_i)$$

$$b_i = \pi D/\lambda \, \sin \theta_i \, (x_2 \cos\phi_i + y_2 \sin\phi_i)$$

$$\cdot$$
$$\cdot$$
$$\cdot$$

$$n_i = \pi D/\lambda \, \sin \theta_i \, (x_N \cos\phi_i + y_N \sin\phi_i)$$

$$i = 1, 2, \ldots, N$$

(A.4)

(Note: in the above notation, "a" refers to array element #1, "b" refers to array element #2, etc.)

Substituting Eq. (A.2) in Eq. (3.2), the covariance matrix in terms of J is

$$\underline{R} = P\underline{\underline{J}}\underline{\underline{J}}^\dagger + \underline{I}$$

(A.5)

where

$$J^\dagger = \begin{bmatrix} e^{-ja_1} & e^{-jb_1} & \cdots & e^{-jn_1} \\ e^{-ja_2} & e^{-jb_2} & \cdots & e^{-jn_2} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ e^{-ja_N} & e^{-jb_N} & \cdot & e^{-jn_N} \end{bmatrix}$$

(A.6)

Only the diagonal terms of Eq. (A.5) are of interest here, which are computed as follows:

$$
\left[
\begin{array}{l}
R_{1,1} = P(e^{j(a_1-a_1)} + e^{j(a_2-a_2)} + \ldots + e^{j(a_N-a_N)}) + 1 = PN+1 \\[2mm]
R_{2,2} = P(e^{j(b_1-b_1)} + e^{j(b_2-b_2)} + \ldots + e^{j(b_N-b_N)}) + 1 = PN+1 \\[2mm]
\qquad \cdot \qquad\qquad\qquad\qquad\qquad\qquad \cdot \\[1mm]
\qquad \cdot \qquad\qquad\qquad\qquad\qquad\qquad \cdot \\[1mm]
\qquad \cdot \qquad\qquad\qquad\qquad\qquad\qquad \cdot \\[2mm]
R_{n,n} = P(e^{j(n_1-n_1)} + e^{j(n_2-n_2)} + \ldots + e^{j(n_N-n_N)}) + 1 = PN+1
\end{array}
\right] \qquad (A.7)
$$

From Eq. (3.4), the sum of the diagonal terms of the covariance matrix is equal to the sum of its eigenvalues, so

$$
\sum_{i=1}^{N} (PN+1) = \sum_{i=1}^{N} \lambda_i \qquad (A.8)
$$

but

$$
\lambda_1 = \lambda_2 = \ldots \lambda_n = \lambda
$$

because it is assumed that N degrees of freedom are completely consumed. Thus,

$$
(PN+1) \sum_{i=1}^{N} 1 = \lambda \sum_{i=1}^{N} 1 \qquad (A.9)
$$

63

which yields

$$\lambda = (PN+1) \qquad\qquad (A.10)$$

as desired. That is, the eigenvalues are proportional to the incident interference power.

REFERENCES

1. A. J. Fenn, "Interference Sources and Degrees of Freedom in Adaptive Nulling Antennas, " Technical Report 604, Lincoln Laboratory, M.I.T. (12 May 1982). AD-A116583

2. W. F. Gabriel, "An Introduction to Adaptive Arrays", Proc. IEEE, 64, 239-271 (1976).

3. J. T. Maynan, "Some Techniques for Evaluating the Bandwidth Characteristics of Adaptive Nulling Systems," IEEE Trans. on Antenna and Propag., AP-27, 373 (1979).

4. J. T. Mayhan, A. J. Simmons, and W. C. Cummings, "Wide-Band Adaptive Antenna Nulling Using Tapped Delay Lines," IEEE Trans. on Antenna and Propag., AP-29, 936 (1981).

5. G. Strang, Linear Algebra and Its Application, (Academic Press, Inc., New York, 1976) p. 213.

6. W. C. Cummings, "Multiple Beam Forming Networks," Technical Note 1978-9, M.I.T. Lincoln Laboratory (18 April 1978). AD-A056904

7. B. Noble and J. W. Daniel, Applied Linear Algebra (Prentice-Hall, New York, 1977).

8. R. L. Zahradnik, Theory and Techniques of Optimization for Practicing Engineers (Barnes and Noble, New York, 1971) pp. 118-124.

9. F. W. Byron, Jr., and R. W. Fuller, Mathematics of Classical and Quantum Physics, 1, (Addison-Wesley Publishing Company, 1969).

10. D. A. Pierre, Optimization Theory with Applications, (J. Wiley and Sons, New York, 1969) pp. 296-299.

11. G. Strang, op. cit., p. 116-117.

12. Ibid; p. 179.

ACKNOWLEDGMENTS

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER ESD-TR-82-089	**2. GOVT ACCESSION NO.** *A122 389*	**3. RECIPIENT'S CATALOG NUMBER**
4. TITLE (and Subtitle) Consumption of Degrees of Freedom in Adaptive Nulling Array Antennas		**5. TYPE OF REPORT & PERIOD COVERED** Technical Report
		6. PERFORMING ORG. REPORT NUMBER Technical Report 609
7. AUTHOR(s) Alan J. Fenn		**8. CONTRACT OR GRANT NUMBER(s)** F19628-80-C-0002
9. PERFORMING ORGANIZATION NAME AND ADDRESS Lincoln Laboratory, M.I.T. P.O. Box 73 Lexington, MA 02173-0073		**10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS** Program Element Nos. 63431F and 33601F Project Nos. 2029/6430
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Systems Command, USAF Andrews AFB Washington, DC 20331		**12. REPORT DATE** 12 October 1982
		13. NUMBER OF PAGES 76
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Electronic Systems Division Hanscom AFB, MA 01731		**15. SECURITY CLASS. (of this report)** Unclassified
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

None

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

phased arrays	eigenvalues	covariance matrix
antennas	interference	gradient search
adaptive nulling	degrees of freedom	

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A gradient search technique is used to maximize consumption of the degrees of freedom for N-channel adaptive nulling phased array antennas. The technique is based on a figure of merit which seeks to maximize the sum of the square roots of the interference covariance matrix eigenvalues. Equivalently, the worst case interference source configuration attempts to completely consume N degrees of freedom of the adaptive nulling antenna. Results are given for several basic regularly spaced arrays. The technique can be applied to a phased array with arbitrary array element positions.

DD FORM 1473 EDITION OF 1 NOV 65 IS OBSOLETE
1 Jan 73

TE
MED
83